# Patera 2005 implementation in Orekit 

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## 1 Demonstration

This note explains the demonstration used to find the Patera2005 method equation used inside Orekit. Indeed, the equation has been modified so that we could use the common input used in short term encounter probability computation $\left(x_{m}, y_{m}, \sigma_{x}, \sigma_{y}, R\right)$. Also, We will only consider the spherical hardbody case here so the radius is constant.

Starting from the second page of Patera's paper (available here) and as we already know the coordinates in the rotated encounter fram ${ }^{1}$. we instead use these coordinates to express the hardbody perimeter :

$$
\begin{align*}
x_{d} & =x_{m}+\rho \cos \theta \\
y_{d} & =y_{m}+\rho \sin \theta \\
\binom{x^{\prime}}{y^{\prime}} & =\left(\begin{array}{ll}
g & 0 \\
0 & 1
\end{array}\right)\binom{x_{d}}{y_{d}} \tag{1}
\end{align*}
$$

With :

$$
\begin{align*}
& \rho=\text { Hardbody radius (constant). } \\
& g=\frac{\sigma_{y}}{\sigma_{x}} \tag{2}
\end{align*}
$$

$$
\theta=\text { Angle from } x_{d} \text { axis to hardbody perimeter point. }
$$

$\triangle$ Different $\theta$ than what is shown in Patera's paper but does not impact the following demonstration (simply start from a different axis).
$₫$ In Patera's paper, $\sigma_{x}^{\prime}$ is equal to the Orekit equivalent $\sigma_{y}$ as Orekit defines the diagonalized combined covariance matrix the other way around ( $\sigma_{x}$ being the smallest covariance axis and $\sigma_{y}$ the largest one).

From there we use similar steps starting from :

$$
\begin{align*}
\tan (\epsilon) & =\frac{y^{\prime}}{x^{\prime}}  \tag{3}\\
& =\frac{y_{m}+\rho \sin \theta}{g x_{m}+g \rho \cos \theta}
\end{align*}
$$

Then we can find :

$$
\begin{align*}
\frac{d \epsilon}{d \theta} & =\frac{d \arctan (\tan (\epsilon))}{d \theta} \\
& =\frac{g \rho\left(x_{m} \cos \theta+y_{m} \sin \theta\right)+g \rho^{2}}{\left(g x_{m}+g \rho \cos \theta\right)^{2}+\left(y_{m}+\rho \sin \theta\right)^{2}}  \tag{4}\\
\frac{d \epsilon}{d \theta} & =\frac{g \rho\left(x_{m} \cos \theta+y_{m} \sin \theta\right)+g \rho^{2}}{r^{2}}
\end{align*}
$$

Finally, we have :

$$
\begin{equation*}
P=\frac{1}{2 \pi} \int_{0}^{2 \pi}\left[1-e^{-\frac{r^{2}}{2 \sigma^{2}}}\right]\left(\frac{g \rho\left(x_{m} \cos \theta+y_{m} \sin \theta\right)+g \rho^{2}}{r^{2}}\right) d \theta \tag{5}
\end{equation*}
$$

With $\sigma=\sigma_{y}$ from the diagonalized combined covariance matrix.

[^0]
[^0]:    ${ }^{1}$ Frame obtained by rotating the initial encounter frame using the rotation matrix used to diagonalize the combined covariance matrix.

