

Analysis of IoD Gooding convergence by Hugo B

Simulation scenario and parameters

Initial date: 1st January 2026

Ground Station position: lat 20.7464 °, Long -156.4314, altitude 3055 m (Maui Optical and Supercomputing observatory)

Orbit types: The test set includes five Keplerian orbits: two LEO (circular at 7 000 km and elliptical at 7 500 km, $e = 0.05$), one MEO at 26 560 km, one GEO at 42 164 km, and one HEO at 26 000 km with $e = 0.7$ and $i = 63.4^\circ$

Observation arc length: Three angles-only observations are taken at times (t_1, t_2, t_3) with total arc durations of 60 s, 120 s, 600 s, 1 200 s and 2 400 s. The middle observation is always centered in the arc.

Initial range guess: The Gooding method takes an initial estimate of the observer-target range at the first and third epochs. This estimate is set as a multiple of the true range, using factors of 0.1, 0.5, 1.0, 2.0, 5.0 and 10.0 to cover both severe underestimates and large overestimates.

Convergence Newton (150 iters max)

Circular Low Earth Orbit

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
LEO_circ	Very short arc	rho_x0.1	0.0	OK
LEO_circ	Very short arc	rho_x0.5	0.3	OK
LEO_circ	Very short arc	rho_x1.0	0.0	OK
LEO_circ	Very short arc	rho_x2.0	0.0	OK
LEO_circ	Very short arc	rho_x5.0	0.0	OK
LEO_circ	Very short arc	rho_x10.0	0.0	OK
LEO_circ	Short arc	rho_x0.1	0.0	OK
LEO_circ	Short arc	rho_x0.5	0.0	OK
LEO_circ	Short arc	rho_x1.0	0.0	OK
LEO_circ	Short arc	rho_x2.0	0.0	OK
LEO_circ	Short arc	rho_x5.0	0.0	OK
LEO_circ	Short arc	rho_x10.0	0.0	OK
LEO_circ	Medium arc	rho_x0.1	0.0	OK
LEO_circ	Medium arc	rho_x0.5	0.0	OK
LEO_circ	Medium arc	rho_x1.0	0.0	OK

LEO_circ	Medium arc	rho_x2.0	0.0	OK
LEO_circ	Medium arc	rho_x5.0	0.0	OK
LEO_circ	Medium arc	rho_x10.0	0.0	OK
LEO_circ	Long arc	rho_x0.1	5020032.6	NA
LEO_circ	Long arc	rho_x0.5	X	Fail
LEO_circ	Long arc	rho_x1.0	0.0	OK
LEO_circ	Long arc	rho_x2.0	0.0	OK
LEO_circ	Long arc	rho_x5.0	0.0	OK
LEO_circ	Long arc	rho_x10.0	X	Fail
LEO_circ	Very long arc	rho_x0.1	X	Fail
LEO_circ	Very long arc	rho_x0.5	0.0	OK
LEO_circ	Very long arc	rho_x1.0	0.0	OK
LEO_circ	Very long arc	rho_x2.0	X	Fail
LEO_circ	Very long arc	rho_x5.0	X	Fail
LEO_circ	Very long arc	rho_x10.0	X	Fail

Elliptical Low Earth Orbit (e=0.05)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
LEO_circ	Very short arc	rho_x0.1	0.0	OK
LEO_circ	Very short arc	rho_x0.5	0.3	OK
LEO_elliptic	Very short arc	rho_x1.0	0.0	OK
LEO_elliptic	Very short arc	rho_x2.0	0.0	OK
LEO_elliptic	Very short arc	rho_x5.0	0.0	OK
LEO_elliptic	Very short arc	rho_x10.0	0.0	OK
LEO_elliptic	Short arc	rho_x0.1	0.0	OK
LEO_elliptic	Short arc	rho_x0.5	0.0	OK
LEO_elliptic	Short arc	rho_x1.0	0.0	OK
LEO_elliptic	Short arc	rho_x2.0	0.0	OK
LEO_elliptic	Short arc	rho_x5.0	0.0	OK
LEO_elliptic	Short arc	rho_x10.0	0.0	OK
LEO_elliptic	Medium arc	rho_x0.1	0.0	OK
LEO_elliptic	Medium arc	rho_x0.5	0.0	OK
LEO_elliptic	Medium arc	rho_x1.0	0.0	OK
LEO_elliptic	Medium arc	rho_x2.0	0.0	OK
LEO_elliptic	Medium arc	rho_x5.0	0.0	OK
LEO_elliptic	Medium arc	rho_x10.0	0.0	OK
LEO_elliptic	Long arc	rho_x0.1	0.0	OK
LEO_elliptic	Long arc	rho_x0.5	0.0	OK
LEO_elliptic	Long arc	rho_x1.0	0.0	OK
LEO_elliptic	Long arc	rho_x2.0	0.0	OK
LEO_elliptic	Long arc	rho_x5.0	0.0	OK
LEO_elliptic	Long arc	rho_x10.0	0.0	OK
LEO_elliptic	Very long arc	rho_x0.1	X	Fail

LEO_elliptic	Very long arc	rho_x0.5	X	Fail
LEO_elliptic	Very long arc	rho_x1.0	0.0	OK
LEO_elliptic	Very long arc	rho_x2.0	0.0	OK
LEO_elliptic	Very long arc	rho_x5.0	X	Fail
LEO_elliptic	Very long arc	rho_x10.0	X	Fail

Medium Earth Orbit (MEO)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
MEO	Very short arc	rho_x0.1	88.1	OK
MEO	Very short arc	rho_x0.5	52.8	OK
MEO	Very short arc	rho_x1.0	17.3	OK
MEO	Very short arc	rho_x2.0	102.7	OK
MEO	Very short arc	rho_x5.0	59.6	OK
MEO	Very short arc	rho_x10.0	64.5	OK
MEO	Short arc	rho_x0.1	0.9	OK
MEO	Short arc	rho_x0.5	0.3	OK
MEO	Short arc	rho_x1.0	1.7	OK
MEO	Short arc	rho_x2.0	19.5	OK
MEO	Short arc	rho_x5.0	5.9	OK
MEO	Short arc	rho_x10.0	6.1	OK
MEO	Medium arc	rho_x0.1	0.0	OK
MEO	Medium arc	rho_x0.5	0.0	OK
MEO	Medium arc	rho_x1.0	0.0	OK
MEO	Medium arc	rho_x2.0	0.1	OK
MEO	Medium arc	rho_x5.0	0.0	OK
MEO	Medium arc	rho_x10.0	0.0	OK
MEO	Long arc	rho_x0.1	0.0	OK
MEO	Long arc	rho_x0.5	0.0	OK
MEO	Long arc	rho_x1.0	0.0	OK
MEO	Long arc	rho_x2.0	0.0	OK
MEO	Long arc	rho_x5.0	0.0	OK
MEO	Long arc	rho_x10.0	0.0	OK
MEO	Very long arc	rho_x0.1	0.0	OK
MEO	Very long arc	rho_x0.5	0.0	OK
MEO	Very long arc	rho_x1.0	0.0	OK
MEO	Very long arc	rho_x2.0	0.0	OK
MEO	Very long arc	rho_x5.0	0.0	OK
MEO	Very long arc	rho_x10.0	0.0	OK

Geostationary Orbit (GEO)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
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GEO	Very short arc	rho_x0.1	260.7	OK
GEO	Very short arc	rho_x0.5	156.4	OK
GEO	Very short arc	rho_x1.0	209.2	OK
GEO	Very short arc	rho_x2.0	14191.6	OK
GEO	Very short arc	rho_x5.0	160.3	OK
GEO	Very short arc	rho_x10	165.4	OK
GEO	Short arc	rho_x0.1	82.8	OK
GEO	Short arc	rho_x0.5	28.4	OK
GEO	Short arc	rho_x1.0	7.3	OK
GEO	Short arc	rho_x2.0	4128.4	OK
GEO	Short arc	rho_x5.0	31.1	OK
GEO	Short arc	rho_x10	4113.9	OK
GEO	Medium arc	rho_x0.1	0.0	OK
GEO	Medium arc	rho_x0.5	0.1	OK
GEO	Medium arc	rho_x1.0	0.1	OK
GEO	Medium arc	rho_x2.0	0.0	OK
GEO	Medium arc	rho_x5.0	0.0	OK
GEO	Medium arc	rho_x10	0.0	OK
GEO	Long arc	rho_x0.1	0.7	OK
GEO	Long arc	rho_x0.5	0.0	OK
GEO	Long arc	rho_x1.0	0.0	OK
GEO	Long arc	rho_x2.0	0.0	OK
GEO	Long arc	rho_x5.0	0.0	OK
GEO	Long arc	rho_x10	0.0	OK
GEO	Very long arc	rho_x0.1	0.0	OK
GEO	Very long arc	rho_x0.5	0.0	OK
GEO	Very long arc	rho_x1.0	0.0	OK
GEO	Very long arc	rho_x2.0	0.0	OK
GEO	Very long arc	rho_x5.0	0.0	OK
GEO	Very long arc	rho_x10	0.0	OK

Highly eccentric orbit (HEO)

HEO	Very short arc	rho_x0.1	15699461.7	NA
HEO	Very short arc	rho_x0.5	0.1	OK
HEO	Very short arc	rho_x1.0	0.0	OK
HEO	Very short arc	rho_x2.0	15699427.5	NA
HEO	Very short arc	rho_x5.0	22464611.6	NA
HEO	Very short arc	rho_x10	22464600.8	NA
HEO	Short arc	rho_x0.1	15716543.3	NA
HEO	Short arc	rho_x0.5	0.0	OK
HEO	Short arc	rho_x1.0	0.0	OK
HEO	Short arc	rho_x2.0	15716555.1	NA
HEO	Short arc	rho_x5.0	X	FAIL
HEO	Short arc	rho_x10	X	FAIL

HEO	Medium arc	rho_x0.1	30516273.2	NA
HEO	Medium arc	rho_x0.5	0.0	OK
HEO	Medium arc	rho_x1.0	0.0	OK
HEO	Medium arc	rho_x2.0	30516811.6	NA
HEO	Medium arc	rho_x5.0	N/A	FAIL
HEO	Medium arc	rho_x10	N/A	FAIL
HEO	Long arc	rho_x0.1	178172258.8	NA
HEO	Long arc	rho_x0.5	0.0	OK
HEO	Long arc	rho_x1.0	0.0	OK
HEO	Long arc	rho_x2.0	0.0	OK
HEO	Long arc	rho_x5.0	X	FAIL
HEO	Long arc	rho_x10	X	FAIL
HEO	Very long arc	rho_x0.1	0.0	OK
HEO	Very long arc	rho_x0.5	0.0	OK
HEO	Very long arc	rho_x1.0	0.0	OK
HEO	Very long arc	rho_x2.0	0.0	OK
HEO	Very long arc	rho_x5.0	0.0	OK
HEO	Very long arc	rho_x10	0.0	OK

Convergence Halley (150 iters max)

Circular Low Earth Orbit

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
LEO_circ	Very short arc	rho_x0.1	0.0	OK
LEO_circ	Very short arc	rho_x0.5	0.0	OK
LEO_circ	Very short arc	rho_x1.0	0.0	OK
LEO_circ	Very short arc	rho_x2.0	0.0	OK
LEO_circ	Very short arc	rho_x5.0	3797570.8	OK
LEO_circ	Very short arc	rho_x10	0.0	OK
LEO_circ	Short arc	rho_x0.1	0.0	OK
LEO_circ	Short arc	rho_x0.5	0.0	OK
LEO_circ	Short arc	rho_x1.0	0.0	OK
LEO_circ	Short arc	rho_x2.0	0.0	OK
LEO_circ	Short arc	rho_x5.0	0.0	OK
LEO_circ	Short arc	rho_x10	0.0	OK
LEO_circ	Medium arc	rho_x0.1	0.0	OK
LEO_circ	Medium arc	rho_x0.5	0.0	OK
LEO_circ	Medium arc	rho_x1.0	0.0	OK
LEO_circ	Medium arc	rho_x2.0	0.0	OK
LEO_circ	Medium arc	rho_x5.0	0.0	OK
LEO_circ	Medium arc	rho_x10	0.0	OK
LEO_circ	Long arc	rho_x0.1	0.0	OK

LEO_circ	Long arc	rho_x0.5	0.0	OK
LEO_circ	Long arc	rho_x1.0	0.0	OK
LEO_circ	Long arc	rho_x2.0	0.0	OK
LEO_circ	Long arc	rho_x5.0	0.0	OK
LEO_circ	Long arc	rho_x10	0.0	OK
LEO_circ	Very long arc	rho_x0.1	0.0	OK
LEO_circ	Very long arc	rho_x0.5	0.0	OK
LEO_circ	Very long arc	rho_x1.0	0.0	OK
LEO_circ	Very long arc	rho_x2.0	0.0	OK
LEO_circ	Very long arc	rho_x5.0	0.0	OK
LEO_circ	Very long arc	rho_x10	0.0	OK

Elliptical Low Earth Orbit (e=0.05)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
LEO_ellip	Very short arc	rho_x0.1	4309771.9	OK
LEO_ellip	Very short arc	rho_x0.5	0.0	OK
LEO_ellip	Very short arc	rho_x1.0	0.0	OK
LEO_ellip	Very short arc	rho_x2.0	0.0	OK
LEO_ellip	Very short arc	rho_x5.0	4347132.6	OK
LEO_ellip	Very short arc	rho_x10	0.0	OK
LEO_ellip	Short arc	rho_x0.1	0.0	OK
LEO_ellip	Short arc	rho_x0.5	0.0	OK
LEO_ellip	Short arc	rho_x1.0	0.0	OK
LEO_ellip	Short arc	rho_x2.0	0.0	OK
LEO_ellip	Short arc	rho_x5.0	4314792.6	OK
LEO_ellip	Short arc	rho_x10	0.0	OK
LEO_ellip	Medium arc	rho_x0.1	0.0	OK
LEO_ellip	Medium arc	rho_x0.5	0.0	OK
LEO_ellip	Medium arc	rho_x1.0	0.0	OK
LEO_ellip	Medium arc	rho_x2.0	0.0	OK
LEO_ellip	Medium arc	rho_x5.0	0.0	OK
LEO_ellip	Medium arc	rho_x10	0.0	OK
LEO_ellip	Long arc	rho_x0.1	0.0	OK
LEO_ellip	Long arc	rho_x0.5	0.0	OK
LEO_ellip	Long arc	rho_x1.0	0.0	OK
LEO_ellip	Long arc	rho_x2.0	0.0	OK
LEO_ellip	Long arc	rho_x5.0	0.0	OK
LEO_ellip	Long arc	rho_x10	0.0	OK
LEO_ellip	Very long arc	rho_x0.1	0.0	OK
LEO_ellip	Very long arc	rho_x0.5	0.0	OK
LEO_ellip	Very long arc	rho_x1.0	0.0	OK
LEO_ellip	Very long arc	rho_x2.0	0.0	OK
LEO_ellip	Very long arc	rho_x5.0	0.0	OK

LEO_ellip	Very long arc	rho_x10	0.0	OK
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Medium Earth Orbit (MEO)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
MEO	Very short arc	rho_x0.1	482131595.2	NOK
MEO	Very short arc	rho_x0.5	82089265806.9	NOK
MEO	Very short arc	rho_x1.0	17.3	OK
MEO	Very short arc	rho_x2.0	1466343011.7	NOK
MEO	Very short arc	rho_x5.0	N/A	FAIL
MEO	Very short arc	rho_x10	12533509550.8	NOK
MEO	Short arc	rho_x0.1	32860412.0	NOK
MEO	Short arc	rho_x0.5	1.6	OK
MEO	Short arc	rho_x1.0	1.7	OK
MEO	Short arc	rho_x2.0	3.6	OK
MEO	Short arc	rho_x5.0	4757394303.7	NOK
MEO	Short arc	rho_x10	89475477.2	NOK
MEO	Medium arc	rho_x0.1	0.0	OK
MEO	Medium arc	rho_x0.5	0.0	OK
MEO	Medium arc	rho_x1.0	0.0	OK
MEO	Medium arc	rho_x2.0	0.0	OK
MEO	Medium arc	rho_x5.0	0.0	OK
MEO	Medium arc	rho_x10	0.0	OK
MEO	Long arc	rho_x0.1	0.0	OK
MEO	Long arc	rho_x0.5	0.0	OK
MEO	Long arc	rho_x1.0	0.0	OK
MEO	Long arc	rho_x2.0	0.0	OK
MEO	Long arc	rho_x5.0	0.0	OK
MEO	Long arc	rho_x10	0.0	OK
MEO	Very long arc	rho_x0.1	0.0	OK
MEO	Very long arc	rho_x0.5	0.0	OK
MEO	Very long arc	rho_x1.0	0.0	OK
MEO	Very long arc	rho_x2.0	0.0	OK
MEO	Very long arc	rho_x5.0	0.0	OK
MEO	Very long arc	rho_x10	0.0	OK

Geostationary Orbit (GEO)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
GEO	Very short arc	rho_x0.1	9245732629.5	NOK
GEO	Very short arc	rho_x0.5	359.4	OK
GEO	Very short arc	rho_x1.0	209.2	OK
GEO	Very short arc	rho_x2.0	285.6	OK

GEO	Very short arc	rho_x5.0	8986831191.8	NOK
GEO	Very short arc	rho_x10	285573909.8	NOK
GEO	Short arc	rho_x0.1	6.6	OK
GEO	Short arc	rho_x0.5	55.6	OK
GEO	Short arc	rho_x1.0	7.3	OK
GEO	Short arc	rho_x2.0	20.0	OK
GEO	Short arc	rho_x5.0	38982980.9	NOK
GEO	Short arc	rho_x10	31820123.2	NOK
GEO	Medium arc	rho_x0.1	0.0	OK
GEO	Medium arc	rho_x0.5	0.1	OK
GEO	Medium arc	rho_x1.0	0.1	OK
GEO	Medium arc	rho_x2.0	0.0	OK
GEO	Medium arc	rho_x5.0	0.0	OK
GEO	Medium arc	rho_x10	0.0	OK
GEO	Long arc	rho_x0.1	0.0	OK
GEO	Long arc	rho_x0.5	0.0	OK
GEO	Long arc	rho_x1.0	0.0	OK
GEO	Long arc	rho_x2.0	0.0	OK
GEO	Long arc	rho_x5.0	0.0	OK
GEO	Long arc	rho_x10	0.1	OK
GEO	Very long arc	rho_x0.1	0.0	OK
GEO	Very long arc	rho_x0.5	0.0	OK
GEO	Very long arc	rho_x1.0	0.0	OK
GEO	Very long arc	rho_x2.0	0.0	OK
GEO	Very long arc	rho_x5.0	0.0	OK
GEO	Very long arc	rho_x10	0.0	OK

Highly eccentric orbit (HEO)

Orbit type	Arc length	Rho factor	Error on SMA (in m)	status
HEO	Very short arc	rho_x0.1	22814271.3	NOK
HEO	Very short arc	rho_x0.5	0.0	OK
HEO	Very short arc	rho_x1.0	0.0	OK
HEO	Very short arc	rho_x2.0	2.5	OK
HEO	Very short arc	rho_x5.0	22726200.4	NOK
HEO	Very short arc	rho_x10	1.1	OK
HEO	Short arc	rho_x0.1	0.0	OK
HEO	Short arc	rho_x0.5	0.0	OK
HEO	Short arc	rho_x1.0	0.0	OK
HEO	Short arc	rho_x2.0	0.0	OK
HEO	Short arc	rho_x5.0	0.0	OK
HEO	Short arc	rho_x10	0.0	OK
HEO	Medium arc	rho_x0.1	0.0	OK
HEO	Medium arc	rho_x0.5	0.0	OK
HEO	Medium arc	rho_x1.0	0.0	OK

HEO	Medium arc	rho_x2.0	0.0	OK
HEO	Medium arc	rho_x5.0	0.0	OK
HEO	Medium arc	rho_x10	0.0	OK
HEO	Long arc	rho_x0.1	0.0	OK
HEO	Long arc	rho_x0.5	0.0	OK
HEO	Long arc	rho_x1.0	0.0	OK
HEO	Long arc	rho_x2.0	0.0	OK
HEO	Long arc	rho_x5.0	0.0	OK
HEO	Long arc	rho_x10	0.0	OK
HEO	Very long arc	rho_x0.1	0.0	OK
HEO	Very long arc	rho_x0.5	0.0	OK
HEO	Very long arc	rho_x1.0	0.0	OK
HEO	Very long arc	rho_x2.0	0.0	OK
HEO	Very long arc	rho_x5.0	22689202.8	NOK
HEO	Very long arc	rho_x10	0.0	OK